HOMEWORK 4

Instruction: Please complete the first 10 problems. The last 5 problems are for you to practice, they are candidates for tests. Please print the homework and work on it directly(It is up to you whether to print it double sided or not). Due on Friday, Feb 10

Problem 1. Show that if $a \equiv b \pmod{m_1}$ and $a \equiv b \pmod{m_2}$, where $\gcd(m_1, m_2) = 1$, then $a \equiv b \pmod{m_1 m_2}$.

Problem 2. Show that if $a^2 \equiv b^2 \pmod{p}$, where p is prime, then either p divides $a+b$ or p divides $a-b$.

Problem 3. Find all integer x such that $-100 \leq x \leq 100$, and $x \equiv 7 \pmod{19}$.

Problem 4. Show that 7 divides $1941^{1963} + 1963^{1991}$.

Problem 5. Determine the last two digits of 9^{9^9}.

Problem 6. Show that 7 divides $5^{2n} + 3 \cdot 2^{5n-2}$ for any positive integer n. (Do not use induction).

Problem 7. Let ϕ be the Euler-Fermat function, show that $\phi(n^2) = n\phi(n)$.

Problem 8. Show that if $n = 2^{2k+1}, \quad k \geq 1$, then $\phi(n)$ is square.

Problem 9. If p isprime then show that $1 + \phi(p) + \phi(p^2) + \cdots + \phi(p^n) = p^n$.

Problem 10. Let a_n is the Fibonacci number, show that $\gcd(a_n, a_{n+1}) = 1$.

Problem 11. Let $R_n = (10^n - 1)/9$, show that $9 \mid R_n$ if and only if $9 \mid n$.

Problem 12. Show that if 7 divides $100a + b$, then 7 divides $2a + b$. Is the converse true?

Problem 13. Find $\phi(n)$ for the following values of n: 100, 200, 7456.

Problem 14. Prove that if $a \equiv b \pmod{m}$, then $\gcd(a, m) = \gcd(b, m)$.

Problem 15. For any positive integer n, show that $\operatorname{lcm}(9n+8, 6n+5) = 54n^2+93n+40$.