HOMEWORK 9

Instruction: Please complete the first 10 problems. Please print the homework and work on it directly(It is up to you whether to print it double sided or not). Due on Monday, April 4

Problem 1. Let X be a discrete random variable and a, b two constants, show that

$$E[aX + b] = aE[X] + b.$$

and

$$Var(aX) = a^2 Var(X).$$

Problem 2. If $E[X] = 1$ and $Var(X) = 5$, find

(1) $E[(2 + X)^2]$
(2) $Var(4 + 3X)$.

Problem 3. Let X be a random variable with parameter (n, p). We are going to find k to maximize $P(X = k)$.

(1) Compute $\frac{P(X=k)}{P(X=k+1)}$;

(2) $P(X = k)$ is maximal if and only if $\frac{P(X=k)}{P(X=k-1)} > 1$ and $\frac{P(X=k+1)}{P(X=k)} < 1$. Find such k.

Problem 4. How many times would you expect to roll a fair die before all 6 sides appeared at least one? (recall coupon collecting problem)

Problem 5. Cards from an ordinary deck are turned face up one at a time. Compute the expected number of cards that need be turned face up in order to obtain

(1) 2 aces;
(2) 5 spades;
(3) all 13 heart.

Problem 6. A deck of n cards, numbered 1 through n, is thoroughly shuffled so that all possible $n!$ ordering can be assumed to be qually likely. Suppose you are to make n guesses sequentially, where the ith one is a guess of the card in position i. Let N denote the number of correct guesses.

(1) If you are not given any information about your earlier guesses. Show that, for any strategy, $E[N] = 1$.
(2) Suppose that after each guess you are shown the card that was in the position in question. What do you think is the best strategy? Show that under this strategy,
$$E[N] = \frac{1}{n} + \frac{1}{n-1} + \cdots + 1.$$

(Hint: For both parts, express N as the sum of indicator random variables.)

Problem 7. Let X be a binomial random variable with parameters (n, p). Compute $E[X]$ by expressing X as the sum of indicator random variables.

Problem 8. Find the expected value and variance of the number of times one must throw a die until the outcome 1 has occurred 4 times

Problem 9. Suppose that X takes on one of the values $0, 1, 2$. If for some constant c, $P(X = i) = cP(X = i - 1)$ for $i = 1, 2$, find $E[X]$.

Problem 10. A communication system consists of n components, each of which will, independently, function with probability p. The total system will be able to operate effectively if at least one-half of its components function. For what value of p is a 5-component system more likely to operate effectively that a 3-component system.