

Applied Combinatorics, Section N2

Test 1

Name: _____

GTID: _____

<i>Problem</i>	<i>Points</i>
1	
2	
3	
4	
5	
6	

TOTAL: _____

Please do show all your work including intermediate steps and also explain in words how you solve a problem. Partial credits are available.

Problem 1. Show inductively that $2^n > n^3$ for $n \geq 10$.

Problem 2. Find all $n \in \mathbf{Z}^+$ such that n divides $5n + 18$.

Problem 3. Solve the following nonhomogenous recurrence relation.

$$a_n = 6a_{n-1} - 9a_{n-2} + 2^n, \quad a_0 = 1, a_1 = 5.$$

Problem 4. Let $a_n = 2^n + n + 1$. Find the generating function of a_n .

Problem 5. Compute $(1000)_2 + (2303)_5$ (Express the result in base 16).

Problem 6. Let $a_n = \lfloor (5 + \sqrt{21})^n \rfloor + 1$, here $\lfloor x \rfloor$ is the floor function, i.e. the largest integer that is less than x . So $a_0 = 2, a_1 = 10, a_2 = 92$.

- (1) Let $b_n = (5 + \sqrt{21})^n + (5 - \sqrt{21})^n$. Find a second order linear recurrence relation that b_n satisfies;
- (2) Show inductively (strong form) that b_n is an integer for any n ;
- (3) Show that $a_n = b_n$ for any $n \geq 0$.