MATH 2602, Midterm I, June 7, 2011

Name: ______________________ **GTID:**_____________

Section: ____________

Problem	*Points*
1	
2	
3	
4	
5	
6	
7	

TOTAL:_____________

Please do show all your work including intermediate steps and also explain in words how you solve a problem. Partial credits are available.

Problem 1 (15pt)**.** The jump form is yet another induction scheme. It works in the following way: Given a statement $P(n)$,

(1) $P(1)$ and $P(2)$ are both true;
(2) $P(k)$ is true implies $P(k+2)$ is true;

Then $P(n)$ is true for any $n \geq 1$.
Use the jump form induction to show the following:

$$1^2 - 2^2 + 3^2 - \cdots + (-1)^{n-1} n^2 = (-1)^{n-1}(1 + 2 + \cdots + n).$$

Problem 2 (15pt)**.** Use induction to prove the following:

$$\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > \sqrt{n}$$

for any $n \geq 2$.

Problem 3 (15pt)**.** Let $a_n, n \geq 1$ be a geometric sequence. Assume $a_4 = 8$ and the product of the first five terms is 1024. Find the first term a and common ratio r.

Problem 4 (5pt+8pt+8pt)**.** Consider the following recurrence relation:

$$a_n = 3a_{n-1} - 2a_{n-2} + 2^n, \quad a_0 = 1, a_1 = 1.$$

(1) Find the roots to the characteristic polynomial. What are the multiplicities of the two roots?
(2) Find a particular solution of this relation.
(3) Solve this recurrence relation.

Problem 5 (9pt+10pt)**.** How many intergers between 1 and 200 are

(1) divisible by 7,11 but not 13?

(2) divisible by exactly two of 7,11 and 13?

Problem 6 (15pt)**.** 101 numbers are chosen from the set $\{1, 2, 3, \ldots, 200\}$. Show that one must be a multiple of another.
Hint: Any natural number can be written in the form $2^k a$ with $k \geq 0$ and a odd.

Problem 7 (Bonus, 15pt)**.** Using generating functions, solve the following third-order recurrence relation:

$$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}, \quad n \geq 3, \quad a_0 = 1, a_1 = 3, a_2 = 6.$$