MATH 2602, Midterm II, June 7, 2011

Name: ______________________________ **GTID:**_________________

Section: ________________

Problem	*Points*
1	
2	
3	
4	
5	
6	
7	

TOTAL:_________________

Please do show all your work including intermediate steps and also explain in words how you solve a problem. Partial credits are available.

Problem 1. (15pt) Determine whether the follwing statement are true or false

[ ] A general graph is Eulerian if and only if every vertex of the graph is even.

[ ] A Hamiltonian graph is Eulerian, but an Eulerian graph is not necessarily Hamiltonian.

[ ] $\mathcal{K}_4$ is a planar graph.

[ ] Any edge added to a tree must produce a cycle.

[ ] Every tree has at least two leaves.

[ ] $\binom{2n}{n} \leq 4^n$ for any $n \geq 1$.

[ ] The number of ways of putting eigth marbles, all of the same color, into 11 numbered boxex is $\binom{18}{8}$.

[ ] A graph that contains a proper cycle cannot be Hamiltonian.

[ ] If A is the adjacency matrix of the graph $\mathcal{K}_5$, then the $(2,4)$ entry of A^2 is 4.

[ ] Any connected subgraph of a tree is also a tree.

Problem 2. (5+5+5+5)A die is tossed ten times and the sequence of the outcomes is observed.

(1) How many different sequences are possible?
(2) How many of these sequences contain a row of exactly three 4's?
(3) How many of these sequences contain at most two 1's?
(4) How many of these sequences have sum equal to 13?

Problem 3. (20pt)

(1) What is the coefficient of x^6 in the binomial expansion of

$$(\frac{3}{x} + x^2)^{18}.$$

(2) Show that

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

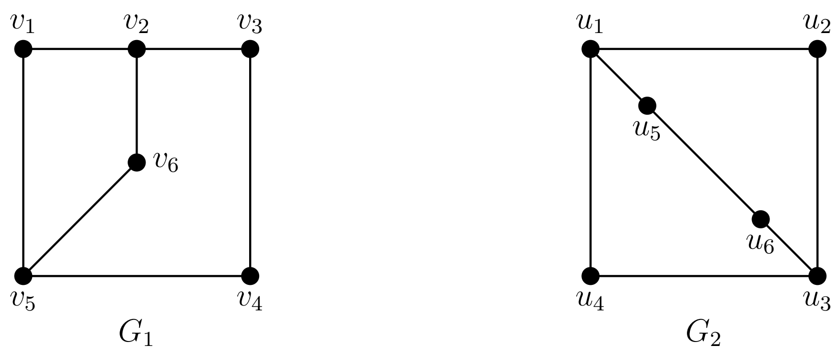
using the fact that

$$\sum_{k=r}^{n} \binom{k}{r} = \binom{n+1}{r+1}$$

for any r, n with $1 \leq r \leq n$.

Problem 4. (20pt) Given the following two graphs G_1 and G_2,

(1) Find the adjacency matrices of G_1;
(2) Explain why G_1 is not Hamiltonian;
(3) Explain why G_2 is not Eulerian;
(4) Show that G_1 and G_2 are isomorphic.



Problem 5. (10pt) In a group of $2n$ people, each person has at least n friends. Show that the group can be seated in a circle, each person next to a friend. (Hint: use Dirac's theorem)

Problem 6. (15pt) Use two different methods to prove

$$\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}.$$