

HOMEWORK 1

Instruction: Please complete the first 10 problems. The last 5 problems are for you to practice, they are candidates for tests. Please print the homework and work on it directly. Due on Wednesday, Jan 23.

Problem 1. Show that

$$\binom{n}{k} = \frac{n}{k} \binom{n-1}{k-1}$$

Problem 2. Show that

$$\sum_{k=0}^n k^2 \binom{n}{k} = (n + n^2)2^{n-2}$$

by taking the second derivative of the expansion of $(1 + x)^n$.

Problem 3. Determine the number of nonnegative integer solutions to the following equations

$$x_1 + x_2 + x_3 = 6.$$

What if the equation is

$$x_1 + x_2 + x_3 \leq 6.$$

Problem 4. Find the coefficient of x^3 in $(x + 2/x^3)^9$.

Problem 5. A die is tossed ten times and the sequence of the outcomes is observed.

- (1) How many different sequences are possible?
- (2) How many of these sequences contain at most 2 1's?
- (3) How many of these sequences have sum equal to 13?

Problem 6. Sixteen people are to be seated at two circular tables, one of which seats 10 while the other seats 6. How many different seating arrangements are possible?

Problem 7. How many positive integers n can be formed using the digits 3,4,4,5,5,6,7 if we want n to exceed 5000000.

Problem 8. Use the binomial theorem to show

$$\sum_{i=0}^n (-1)^i \binom{n}{i} = 0.$$

Problem 9. A committee of 6 people is to be chosen from a group consisting of 7 men and 8 women. If the committee must consist of at least 3 women and at least 2 men. How many different committees are possible.

Problem 10. Write a MATLAB function to generate 10 uniformly distributed numbers in interval $[2, 4]$ using rand function.

Problem 11. Expand $(3x^2 + y)^5$.

Problem 12. Show that

$$\sum_{k=0}^n k^3 = \left(\frac{n(n+1)}{2} \right)^2$$

by using the following fact

$$\sum_{k=r}^n \binom{k}{r} = \binom{n+1}{r+1}$$

for any r, n with $1 \leq r \leq n$

Problem 13. In how many ways can one toss 100 dice so that all 6 types of face will be showing.

Problem 14. In how many ways can 17 be written as a sum of 2's and 3's if the order of the summands is not relevant.

Problem 15. In how many ways can a man divide 7 gifts among his 3 children if the eldest is to receive 3 gifts and the others 2 each?