HOMEWORK 2

Instruction: Please complete the first 10 problems. The last 5 problems are for you to practice, they are candidates for tests. Please print the homework and work on it directly. Due on Monday, Feb 4.

Problem 1. Expand $\mathbf{P}(E_1 \cup E_2 \cup E_3 \cup E_4)$.

Problem 2. Prove the following relations

(1) $EF \subset E \subset E \cup F$.
(2) If $E \subset F$, then $F^c \subset E^c$.
(3) $F = FE \cup FE^c$.

Problem 3. Let E, F, G be three events. Find expressions for the events so that, of E, F, G,

(1) only E occurs;
(2) both E and G, but not F, occur;
(3) at least two of the events occur;
(4) at most two of the events occur;
(5) all three events occur;
(6) none of the events occur;
(7) exactly two of the events occur;

Problem 4. Suppose an experiment is performed n times. For any event E of the sample space, let $n(E)$ denote the number of times that event E occurs and define $f(E) = n(E)/n$. Show that $f(\cdot)$ satisfies three axioms of probability. In other words, f is a probability function.

Problem 5. For any two events E and F, prove the following inequality

$$P(EF) \geq P(E) + P(F) - 1.$$

Problem 6. A pair of fair dice is rolled. What is the probability that the second die lands on a higher value than does the first?

Problem 7. A die is rolled until 2 or 5 occurs. Find the probability that 2 occurs first.(Hint: Let E_n be the event that 2 occurs on the nth roll for the first time and no 5 occurs before. Then compute $\mathbf{P}(\cup_i^\infty E_i)$).

Problem 8. A box contains 3 red and 7 black balls. Player A and B withdraw balls from the box consecutively until a red ball is selected. Find the probability that A selects the red ball. (A draws the first ball, then B, and so on. There is no replacement.)

Problem 9. A closet contains 10 pairs of shoes. If 8 shoes are randomly selected, what is the probability that there will be

(1) no complete pair?
(2) exactly 1 complete pair?

Problem 10. If 4 married couples are arranged in a row, find the probability that no husband sits next to his wife.

Problem 11. A group of individuals containing b boys and g girls is lined up in random order. Each of the $(b+g)!$ permutations is assumed to be equally likely. What is the probability that the person in ith position is a girl?

Problem 12. If a die is rolled 4 times, what is the probability that 6 comes up at least once?

Problem 13. Five balls are randomly chosen, without replacement, from an box that contains 5 red, 6 white and 7 blue balls. Find the probability that at least one ball of each color is chosen.

Problem 14. A box contains n red and m blue balls. They are withdrawn one at a time until a total of $r (r \leq n)$ red balls have been withdrawn. Find the probability that a total of k balls are withdrawn.

Problem 15. Suppose E and F are mutually exclusive events for which $\mathbf{P}(E) = 0.3$ and $\mathbf{P}(F) = 0.5$. What is the probability that

(1) either E or F occurs?
(2) E occurs but F does not?
(3) both E and F occur?