Introduction to Probaility and Statistics

Sample Test 2

Name: ________________________________

GTID: ________________________________

Problem	*Points*
1	
2	
3	
4	
5	
6	

TOTAL:__________________

Please do show all your work including intermediate steps and also explain in words how you solve a problem. Partial credits are available.

Problem 1. There are N distinct types of coupons in cereal boxes and each type, independent of prior selections, is equally likely to be in a box. What is the expected number of distinct coupon types if a child has collected n coupons?

Problem 2. If X is an exponential random variable with parameter $\lambda = 1$, compute the probability density function of the random variable Y defined by $Y = \log X$.

Problem 3. If 65 percent of the population of a large community is in favor of a proposed rise in school taxes, approximate the probability that a random sample of 100 people will contain

(1) at least 50 who are in favor of the proposition;
(2) between 60 and 70 inclusive who are in favor;
(3) fewer taht 75 in favor

Problem 4. (1) Let X_1 be the number of tosses one need to perform before getting 3 or 6 when tossing a fair die.

(2) Let X_2 be the number of tosses one need to perform before accumulating three 1s when tossing a fair die.

(3) Let X_3 be the number of white balls being selected when a sample of size 7 is chosen from a box containing 10 white balls and 20 black balls.

(4) Let X_4 be the number of balls one need to withdraw from a box containing 10 white balls and 20 black balls in order to accumulate 5 white balls.

Identify the distribution of X_1, X_2, X_3, X_4 and find their expectations.

Problem 5. The random variable X has the probability density function

$$f(x) = \begin{cases} ax + bx^2, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

If $E[X] = 0.6$, find

(1) $P(X < 1/2)$;

(2) $\mathrm{Var}(X)$.

Problem 6. Let Q_n denote the probability that no run of 3 consecutive heads appears in n tosses of a fair coin. Show that

$$Q_n = \frac{1}{2}Q_{n-1} + \frac{1}{4}Q_{n-2} + \frac{1}{8}Q_{n-3}$$
$$Q_0 = Q_1 = Q_2 = 1$$

and find Q_8 (Hint: condition on when the first tail occurs).