Introduction to Probaility and Statistics

Sample Test 3

Name: ______________________________

GTID: ______________________________

Problem	*Points*
1	
2	
3	
4	
5	
6	

TOTAL:________________

Please do show all your work including intermediate steps and also explain in words how you solve a problem. Partial credits are available.

Problem 1. Identify the distribution of $X + Y$ if

(1) $X \sim U(0, 2)$, $Y \sim U(0, 2)$.
(2) $X \sim B(m, p)$, $Y \sim B(n, p)$.
(3) $X \sim \text{Poiss}(\lambda_1)$, $Y \sim \text{Poiss}(\lambda_2)$.
(4) $X \sim N(\mu_1, \sigma_1^2)$, $Y \sim N(\mu_2, \sigma_2^2)$.

Problem 2. Let U denote a random variable uniformly distributed over $(0, 1)$. Compute the conditional distribution of U given that $U > a$.

Problem 3. The random variables X and Y have joint density function

$$f(x,y) = \begin{cases} 12xy(1-x), & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(1) Are X and Y independent?
(2) Find $E[X]$.
(3) Find $\text{Var}(X)$.

Problem 4. If X and Y are independent and identically distributed uniform random variables on $(0,1)$, compute the joint density of $U = X+Y$, $V = X/Y$;

Problem 5. The random variables X and Y have a joint density function given by

$$f(x,y) = \begin{cases} 2e^{-2x}/x, & 0 \le x < \infty, 0 \le y \le x \\ 0, & \text{otherwise} \end{cases}$$

Compute $\text{Cov}(X, Y)$.

Problem 6. The moment generating function of X is given by $M_X(t) = \exp(2e^t - 2)$ and that of Y by $M_Y(t) = (\frac{3}{4}e^t + \frac{1}{4})^{10}$. If X and Y are independent, what is $E[XY]$?